

Spin dependent structure function $g_2(x)$ in quark-parton model. Possible interpretation and numerical estimates

N.L.Ter-Isaakyan

Yerevan Physics Institute

Abstract

It is shown that in the special infinite momentum frame where photon has pure transverse components at $P \rightarrow \infty$ the spin-dependent deep inelastic structure function $g_2(x)$ has a reasonable interpretation in terms of quark-parton wave functions, whereas in the conventional frame where photon has pure z-component the parton model fails for $g_2(x)$. The spin dependent structure functions $g_1(x)$ and $g_2(x)$ have been calculated in the relativistic quark model constructed in such frame. The results indicate significant twist-3 contribution.

1 Introduction

Experiments designed to measure the spin-dependent structure functions of the nucleon are now being performed (see, e.g., [1]). As it is well known the structure function $g_2(x)$ has a transparent interpretation in the quark parton model (see, e.g. [2, 3]), whereas the parton interpretation of $g_2(x)$ faces serious difficulties, which are connected with the fact that this function turned out to be zero for a free quark (see e.g., [3]) and, hence, cannot be presented by its value on a free quark averaged over the proper probabilities of parton distributions. Therefore, the nonzero value of $g_2(x)$ can be obtained if we take into account parton interactions (or parton off-shellness). But the results of such calculations depend on the coordinate system (see below) and turn out to be physically unreasonable. So, in the conventional frame where the proton has pure z-component [2, 3]

$$P_\mu = (E, P, 0, 0), \quad q_\mu = (0, -2Px, 0, 0), \quad (1)$$

the value of $g_1(x) + g_2(x)$ vanishes for massless quarks and the Burkhardt - Cottingham sum rule [4],

$$\int_0^1 g_2(x) dx = 0 \quad (2)$$

is not fulfilled.

It is convinient to analyze the parton model considering the old fashioned perturbation theory diagrams in the infinite momentum frame (IMF). In general, when we want to take into account parton interactions, in addition with naive parton model diagram of Fig. 3, the diagrams of fig.2 which contain $q\bar{q}$ -pairs creation or annihilation must be also taken into account.

Fig. 1: The naive parton model diagram of deep inelastic scattering. The arrows denote nucleon and guarks helicities for helicity-flip amplitude which determine structure function $g_2(x)$

Fig. 2: The diagrams, which could violate the parton model for $g_2(x)$.

It is shown in this paper that such diagrams may contribute to $g_2(x)$ and violate the validity of parton model. We show that only in special IMF it is possible to get reed of such diagrams and to represent $g_2(x)$ in terms of quark-parton infinite momentum wave functions. I find that there are two phenomenologically independent

Fig. 3: The diagrams, which determine the contribution to $g_2(x)$ of Compton amplitude with different number of partons in the initial and final nucleon wave functions.

contributions to $g_2(x)$. The first contribution is determined by the diagrams with the different values of quark orbital angular momentum projections in the initial and final nucleon wave functions (fig.1)¹. The second contribution is determined by Compton amplitudes with different number of gluons in the initial and final nucleon wave functions (fig.3). In the field theory these two contributions may be apparently connected.

The similar interpretation of $g_2(x)$ were discussed in Ref.5 in the framework of operator product expansion on "light cone".

In the second part of this paper I calculate the structure functions $g_1(x)$ and $g_2(x)$ in the relativistic quark model of ref.6. The results agree with the bag model calculations [7] and with QCD sum rule result [8, 9] and indicate a significant twist-3 contribution to $g_2(x)$ in the range of $x \leq 0.5$.

2 Quark-parton interpretation of $g_2(x)$

We start from the standard definition of the hadronic tensor:

$$\frac{M}{4\pi} W_{\mu\nu} = \sum_X (2\pi)^4 \langle p, s | J_\mu | X \rangle \langle X | J_\nu | p, s \rangle \delta(P + q - P_X) =$$

$$P_\mu P_\nu W_2 - M^2 W_1 g_{\mu\nu} + iM \epsilon_{\mu\nu\lambda\sigma} q^\lambda [M^2 s^\sigma G_1 + (Pq s^\sigma - sq P^\sigma) G_2] \quad (3)$$

(disregarding terms proportional to q_μ or q_ν), where W_1 , W_2 , G_1 and G_2 are functions of q^2 and $\nu = Pq/M$. The nucleons are supposed in the same spin state described by s^μ . In the coordinate system (1) the spin average structure functions $F_1(x) = MW_1(x)$ and $F_2(x) = \nu W_2(x) = 2xF_2(x)$ are determined by symmetric part of the hadronic tensor W_{ij}^s ($i, j = 1, 2$). The spin dependent structure functions may be expressed through antisymmetric parts of W_{ij} and W_{i0} ($i = 1, 2$) as follows:

$$\frac{1}{2\pi} W_{ij}^a = 2i\epsilon_{ij} g_1(x) \frac{s_0}{2P}$$

¹For simplicity, all diagrams are given for three quark state

$$\frac{1}{2\pi}W_{i0}^a = i\epsilon_{ij}s_j(g_1(x) + g_2(x))\frac{2M}{P} \quad (4)$$

where the functions $g_1 = M^2\nu G_1$ and $g_2 = M\nu^2 G_2$ scales in the Bjorken limit. For quark momenta (which are defined on diagrams) we introduce the standart parameterizations:

$$\vec{p}_1 = x_1\vec{P} + \vec{p}_{1\perp}, \quad \vec{p}'_1 = -x_1\vec{P} + \vec{p}_{1\perp}, \quad \vec{p}''_1 = -x_1\vec{P} - \vec{p}_{1\perp},$$

$$\vec{p}_{1\perp}\vec{P} = 0 \quad (5)$$

In the coordinate system (1) the vertices of photon interactions with quarks (on fig.1) and with $q\bar{q}$ -pairs (on fig.2) at $P \rightarrow \infty$ behave as follows (i,j=1,2):

$$\bar{u}(p'_1)\gamma_i u(p_1) = 2Px_1\sigma_i\sigma_3, \quad \bar{u}(p'_1)\gamma_0 u(p_1) = 2(m + i\epsilon_{ik}\sigma_i p_{1k}),$$

$$\bar{u}(p'_1)\gamma_i v(p''_1) = 2(m\sigma_i + i\epsilon_{ij}p_{1j})\sigma_2, \quad \bar{u}(p'_1)\gamma_0 v(p''_1) = 2Px_1\sigma_3\sigma_2. \quad (6)$$

The amplitude of antiquark interaction with nucleon also may behave as P. Hence, the large energy denominators corresponding to dashed lines on diagrams of fig.2 may be compensated for W_{i0} (but not for W_{ij}) and these diagrams may contribute to $(g_1(x)+g_2(x))$. Thus, the naive parton model fails for $g_2(x)$ in the coordinate system (1). Let us consider now the special IMF where photon have pure transverse component at $P \rightarrow \infty$:

$$P_\mu = (E, P, 0, 0), \quad q_\mu = (\frac{q_\perp^2}{4Px}, \frac{-q_\perp^2}{4Px}, 0, 0). \quad (7)$$

In this frame the structure functions $g_1(x)$ and $g_2(x)$ are expressed through antisymmetric component of the hadronic tensor W_{i0}^a in the following form:

$$\frac{1}{4\pi}W_{i0}^a \frac{q_\perp^2}{x} = 2i\epsilon_{ij}q_j s_0 g_1(x)$$

$$\frac{1}{4\pi}W_{i0}^a \frac{q_\perp^4}{x^2} = 2i\epsilon_{ij}q_j \vec{s} \vec{g}_\perp M g_2(x) \quad (8)$$

We shall take longitudinally polarized nucleon to extract $g_1(x)$ and transversely polarized nucleon in \vec{q}_\perp direction to extract $g_2(x)$. But it is more transparent physically to represent $g_2(x)$ in terms of helicity amplitudes. In that language $g_2(x)$ corresponds to nucleon helicity-flip Compton amplitude. In the IMF (7) the vertices of photon interactions with quarks (on fig.1) and with $q\bar{q}$ -pairs (on fig.2) at $P \rightarrow \infty$ behave as follows (i=1,2):

$$\bar{u}(p'_1)\gamma_0 u(p_1) = 2Px_1, \quad \bar{u}(p'_1)\gamma_i u(p_1) = 2(p_{1i} + q_i + i\epsilon_{ik}q_k\sigma_3),$$

$$\bar{u}(p'_1)\gamma_0 v(p''_1) = \vec{\sigma} \vec{q} \sigma_3 \sigma_2, \quad \bar{u}(p'_1)\gamma_i v(p''_1) = -2Px_1\sigma_i\sigma_2. \quad (9)$$

In the scaling limit the energy δ -function gives:

$$\delta(E + q_0 - E_X) = \frac{2Pxx_1}{q_\perp^2} \delta[x_1 - x(1 + \frac{2\vec{q}\vec{p}_{1\perp}}{q_\perp^2})]. \quad (10)$$

The contributions of diagrams of fig.2 to W_{i0} do not vanish at $P \rightarrow \infty$, this amplitude behaves as $W_{i0} \sim P/q_\perp^2$ in the scaling limit and these diagrams could contribute, in principle, to $g_2(x)$. But it is easy to see that W_{i0} do not depend on \vec{q}_\perp direction and hence cannot contain necessary structure: $\epsilon_{ij}q_j\vec{s}\vec{q}$. That means that such diagrams do not actually contribute to $g_2(x)$ and its correct value can be derived taking into account only diagram of fig.1, if we compare, for instance, terms proportional to q_iq_j at both sides of (8).

At first sight the diagram of fig.1 also do not contribute to $g_2(x)$ because the corresponding value of W_{i0} has a wrong \vec{q}_\perp dependence and do not contain spin-flip terms. To obtain the nonzero value of $g_2(x)$ we have to take into account the second term of the expansion of δ - functions argument at $q_\perp^2 \rightarrow \infty$ in (10). Finally, the contribution of the diagram of Fig.1 may be presented in the form:

$$g_2^{(1)}(x) = \frac{d}{dx}\bar{g}(x),$$

$$2M\bar{g}(x) = \sum_r \int d\Gamma^n \delta(x - x_r) \sum_{s_1, \dots, s_n} \Psi_{s_i}^{*\uparrow}(x_i, \vec{p}_{i\perp}) 2s_r Q_r^2 \Psi_{s_i}^\downarrow(x_i, \vec{p}_{i\perp}) (p_{r\perp}^x + ip_{r\perp}^y), \quad (11)$$

where $d\Gamma^n$ is n-particle phase space:

$$d\Gamma^n = \frac{1}{x_n} \sum_{i=1}^{n-1} \frac{dx_i d\vec{p}_{i\perp}}{2x_i (2\pi)^3}, \quad (12)$$

Q_r and s_r denote charge and spin projection of active quark along z direction. The energy denominators are included into nucleon wave function:

$$\Psi_{s_i}(x_i, \vec{p}_{i\perp}) = \frac{\Gamma_{s_1 \dots s_n}(x_1, \vec{p}_{1\perp} \dots x_n, \vec{p}_{n\perp})}{2P(E - \sum_i E_i)} \quad (13)$$

where $\Gamma_{s_1 \dots s_n}(x_1, \vec{p}_{1\perp} \dots x_n, \vec{p}_{n\perp})$ is the nucleon-partons vertex.

The normalization condition of nucleon wave function (13) can be fixed, for instance, from normalization of nucleon electric formfactor $F_1(Q^2)$ at $Q^2 = 0$ in the quark parton model and has a following form:

$$\int d\Gamma^n \sum_{s_1, \dots, s_n} \Psi_{s_i}^*(x_i, \vec{p}_{i\perp}) \Psi_{s_i}(x_i, \vec{p}_{i\perp}) = 1 \quad (14)$$

(For more details of deriving Egs.12-14 for three quark states see ref.5). In the integral over transverse momentum in (11) only terms which contains linear powers of transverse momenta in the final or initial wave functions contribute. Such terms can arise when the difference of angular orbital momentum projections of initial and final states is equal to unity, $\Delta\langle L_z \rangle = 1$; for such states the sum of parton helicities is not equal to nucleon helicity and hence nucleon spin-flip could take place as it is shown on the diagram (fig.1). The sum rule (2) will be fulfilled for (11)

if $\bar{g}(1) = \bar{g}(0) = 0$. The bound state wave functions vanish at $x=0$ and $x=1$ and the QCD-evolution should not violate the sum rule (2). So, if we suppose that quark-antiquark sea in the nucleon arise due to QCD-evolution and at low resolution scale nucleon can be considered as a bound state of finite number of constituents, the sum rule (2) will be fulfilled for (11).

The structure function $g_1(x)$ in the same notations has a following form:

$$2g_1(x) = \sum_r \int d\Gamma^n \delta(x - x_r) \sum_{s_1, \dots, s_n} \Psi_{s_i}^{*\uparrow}(x_i, \vec{p}_{i\perp}) 2s_r Q_r^2 \Psi_{s_i}^\uparrow(x_i, \vec{p}_{i\perp}). \quad (15)$$

Now consider the diagrams of fig.3. The momenta are defined on the diagrams:

$$\vec{p}_1 = x_1 \vec{P} + \vec{p}_{1\perp}, \quad \vec{k}_1 = y_1 \vec{P} + \vec{k}_{1\perp}, \quad \vec{k}_g = y_g \vec{P} + \vec{k}_{g\perp}, \quad (16)$$

The energy denominators corresponding to dashed lines on diagrams behave at $P \rightarrow \infty$ and $q_\perp \rightarrow \infty$ as

$$\frac{1}{2E_1(E + q_0 - \sum_i E_i)} = \mp \frac{x}{q_\perp^2(x_1 - y_1)}. \quad (17)$$

The upper sign in (16) corresponds to fig.3a, and bottom sign to fig.3b diagrams, respectively. The quark-gluon vertices on these diagrams behave as

$$\bar{u}(p'_1) \gamma_i u(k'_1) = q_i(x_1 + y_1) + i\sigma_3 \epsilon_{ik} q_k(x_1 - y_1) \quad (18)$$

for transverse gluons and these diagrams do contribute to $g_2(x)$ (but do not contribute to other structure functions). The diagrams with $q\bar{q}$ - pairs creations or annihilations (figs. 2c, 2d) do not contain necessary structure $\epsilon_{ij} q_j \vec{s} \vec{q}$ and do not contribute to $g_2(x)$. It is easy to understand that nucleon and gluon spins must be aligned in the same direction and the nucleon spin-flip takes place as it is demonstrated on the diagrams. We find the following result for these diagrams:

$$2Mg_2^{(2)}(x) = \sqrt{2x_r y_r} \sqrt{4\pi\alpha_s} \sum_r \int d\Gamma^n [\delta(x - x_r) - \delta(x - y_r)] \frac{dy_g d\vec{k}_{g\perp}}{(2\pi)^3 2y_r y_g^2} \times \sum_{s_1, \dots, s_n} (\Psi_{s_i}^{*\downarrow}(x_i, \vec{p}_{i\perp}) \Psi_{s_i}^\uparrow(x_i, \vec{p}_{i\perp}) + \Psi_{s_i}^{*\uparrow}(x_i, \vec{p}_{i\perp}) \Psi_{s_i}^\downarrow(x_i, \vec{p}_{i\perp})) 2s_r Q_r^2, \quad (19)$$

Here $\Psi_{s_i\lambda} = 1/2\lambda_a \psi_{s_i\lambda}^a$ (λ_a denotes Gell-Mann matrices) is the wave function of nucleon consisting of n-partons and one transverse gluon with helicity λ . The Burkhardt-Cottingham sum rule (2) is fulfilled for $g_2(x)$ due to cancelation of the contributions of fig.3a and fig.3b diagrams according to (17).

Thus, the structure function $g_2(x)$ in the parton model is determined by two physically independent mechanisms which do not contribute to other structure functions, so measuring $g_2(x)$ we can get essentially new information about quark-gluon structure of the nucleon, namely information about angular orbital momentum distribution and information about gluon distribution in the nucleon.

The direct connection of our results with the results of operator product expansion [5, 7] (see, also, Ref.10 and references therein) is not obvious and needs further

considerations. Note only, that as it easy to understand from Egs.11,15 $g_2^{(1)}(x)$ corresponds to twist-2 and twist-3 contributions, whereas $g_2^{(2)}(x)$ corresponds only to twist-3 contribution. The QCD evolution of $g_2^{(2)}(x)$ is, apparently, more complicated and differs from evolution of $g_2^{(1)}(x)$, so the structure function $g_2(x)$ cannot be evolved from its value at some low Q^2 , in accordance with results of operators product expansion [6, 7].

3 Numerical calculations of $g_1(x)$ and $g_2(x)$ in relativistic quark model

In order to estimate the role of twist-3 contribution and to understand in details the physical interpretation of $g_2(x)$ it is worth to calculate this function in a specific model. In this section the results of such calculation in the relativistic quark model (RQM) of Ref.6 are presented. We actually assume that at low virtualities nucleon consists of three valence constituent quarks only and do not discuss the contributions of diagrams of fig.3. In such a simple model it is impossible to reproduce the the correct Regge behavior at $x \rightarrow 1$. So, our results are expected to be reasonable in the range of not small values of x ($x \leq 0.2 - 0.3$), where the contributions of nonperturbative quark-antiquark pairs are expected to be small. The results obtained must be evaluated into the range of experimental values of Q^2 . In general, it is impossible to write down the simple evolution equation for twist-3 contribution to $g_2(x)$ which would allow to connect this function at different values of Q^2 [10, 11]. Nevertheless in Ref.12 it was found that such approximate equation (which becomes exact in $N \rightarrow \infty$ limit) exist and utilizing this result it is possible to evaluate our results into the range of experimental momentum transfer.

In RQM the spin dependent structure functions $g_1(x)$ and $g_2(x)$ are determined by Egs.11,15 with $n=3$. The nucleon wave functions (13) in the IMF have following form [6]²:

$$\Psi_{s_1 s_2 s_3}^s(x_i, p_{i\perp}) = \Phi(M_0^2) U_{s_1 s'_1}(x_1, p_{1\perp}) U_{s_2 s'_2}(x_2, p_{2\perp}) U_{s_3 s'_3}(x_3, p_{3\perp}) \chi_{s'_1 s'_2 s'_3}^s \quad (20)$$

where $\Phi(M_0^2)$ is a radial part which supposed to depend only of one argument - invariant mass of the system of quarks, composing nucleon [6],

$$M_0^2 = \sum_{i=1}^3 \frac{p_{i\perp}^2 + m^2}{x_i}, \quad (21)$$

where m stands for constituent quark mass; $\chi_{s'_1 s'_2 s'_3}^s$ is the spin orbital part of wave function which is supposed to coincide with nonrelativistic wave functions of naive quark model. The Melosh matrices,

²Different variants of relativization of quark model differ, in general, by explicit form of Melosh matrices, see e.g.,[13] and references therein.

Fig. 4: The results of numerical calculations of $g_2(x)$ and $g_2(x)$ in relativistic quark model.

$$U(x_i, p_{i\perp}) = \frac{m + M_0 x_i + i\epsilon_{mn}\sigma_m(p_{i\perp})_n}{(m + M_0 x_i)^2 + p_{i\perp}^2} \quad (22)$$

determine the transformation of $\chi_{s_1 s_2 s_3}^s$ functions from rest frame into IMF. In Ref.6 a good description of nucleon static parameters were obtained under assumption that nucleon wave function is pure $[56.0^+]$ representation of SU(6) group. For radial wave function it was supposed the following form:

$$\Phi(M_0^2) = N \exp(-M_0^2/6\alpha^2) \quad (23)$$

The fitting parameters, α and m turned out to be [6]:

$$\alpha = 380 \pm 60 \text{ Mev}, m = 270 \pm 30 \text{ Mev} \quad (24)$$

Note, that parameter α characterizes the mean square momentum of quark in nucleon: $p_{\perp}^2 \simeq 2/3\alpha^2$. It is very important to note that due to Melosh transformation in 20 the nucleon IMF wave function depends on quark transvers momentum even for pure $[56.0^-]$ representation. This actually means that that due to Melosh transformation in pure SU(6) wave function in IMF arises the admixture of states with nonzero value of angular orbital momentum. It makes it possible to derive the nonzero value of $g_2(x)$. We find the following results for spin dependent structure functions:

$$2g_1^p(x) = \frac{4Q_u^2 - Q_d^2}{3} \int d\Gamma^3 \delta(x - x_1) \left[1 - \frac{2p_{1\perp}^2}{(m + M_0 x_1)^2 + p_{1\perp}^2} \right] \quad (25)$$

$$2Mg_2^p(x) = \frac{4Q_u^2 - Q_d^2}{3} \frac{d}{dx} \int d\Gamma^3 \delta(x - x_1) \frac{p_{1\perp}^2 (m + M_0 x_1)}{(m + M_0 x_1)^2 + p_{1\perp}^2} \quad (26)$$

For the neutron structure functions the replacement $Q_u \leftrightarrow Q_d$ must be inserted in (25,26) and we find:

$$g_1^n(x) = g_2^n(x) = 0 \quad (27)$$

This result qualitatively agrees with low experimental value of $g_1^n(x)$. The nonzero values of $g_1^n(x)$ and $g_2^n(x)$ can be obtained if we take into account the admixture of high multiplets in the nucleon wave function.

The nucleon β -decay coupling

$$g_A = 6 \int (g_1^p(x) - g_1^n(x)) dx \simeq 1.22 \quad (28)$$

is well described in the model.

The part of nucleon spin carried by quarks in IMF is given by

$$2\langle S_z \rangle = \int d\Gamma^3 [1 - \frac{2p_{1\perp}^2}{(m + M_0 x_1)^2 + p_{1\perp}^2}] \simeq 0.73 \quad (29)$$

and turned out to be less than unity even for pure SU(6) nucleon wave function (see also Ref.14). The missing part of nucleon spin is carried by angular orbital momentum L_z which arise due to Melosh transformations. It is easy to check that $\langle L_z \rangle$ exactly correspond to the second term in square brackets in (30). But this interesting relativistic effect cannot account for the spin deficit which follows from experimental data. Keeping in mind these results (29-30) it is natural to assume that the considered model more successfully can be applied in flavour non-singlet channel. This is, apparently, connected with possible cancelation of unknown contributions which could be essential in singlet channel. So, for comparison with experiment or with predictions of other models it is more reasonable to consider the differences of proton and neutron structure functions.

The results of numerical calculations are presented on Fig.4. The twist-3 contribution to $g_2(x)$ was obtained by subtracting from $g_2(x)$ (27) the twist-2 contribution which is determined by $g_1(x)$ [15]:

$$g_2^{tw.2}(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy \quad (30)$$

Our results are very close to the bag model calculations of Ref.7 and indicate that twist-3 contribution is not small. Remind that these results correspond to low resolution scale, of order of typical constituent quark virtualities in the nucleon. The evolution to the experimental values of Q^2 will increase our predictions considerably (about 5-8 times) in the range of $x \geq 0.5$.

The second moments of twist-3 contribution to $g_2(x)$ were calculated in Refs.8,9 in framework of QCD sum rules:

$$M_{p-n}^{(2)} = \int (g_2^p - g_2^n) x^2 dx = 0.008 \pm 0.004 \quad (31)$$

$$M_{p+n}^{(2)} = \int (g_2^p + g_2^n) x^2 dx = -0.009 \pm 0.004 \quad (32)$$

Our result for difference $g_2^p(x) - g_2^n(x)$

$$M_{p-n}^{(2)} \simeq 0.002 \quad (33)$$

agrees with (31) by sign and by order of magnitude.

4 Conclusion

In the special infinite momentum frame the structure function $g_2(x)$ has a reasonable interpretation in terms of nucleon parton wave function (but not in terms of probabilities). We have two contribution which are phenomenologically independent. The first contribution corresponds to Compton diagrams with different values of angular orbital momenta projections in the initial and final nucleon wave functions. This contribution is proportional to quark transvers momentum and corresponds to twist-2 and twist-3 contributions in terms of OPE. The second contribution is determined by Compton diagrams with different number of gluons in initial and final nucleon wave functions and correspond only to twist-3.

According to independent estimates in different models the function $g_2(x)$ and the twist-3 contribution to $g_2(x)$, which really contains new information on quark-gluon interaction in nucleon are not small. So, the projected measurements of this function are reasonable and will provide new information on nucleon structure.

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N.L.Ter-Isaakyan

Yerevan Physics Institute

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1 Introduction

Experiments designed to measure the spin-dependent structure functions of the nucleon are now being performed (see, e.g., [1]). As it is well known the structure function $g_2(x)$ has a transparent interpretation in the quark parton model (see, e.g. [2, 3]), whereas the parton interpretation of $g_2(x)$ faces serious difficulties, which are connected with the fact that this function turned out to be zero for a free quark (see e.g., [3]) and, hence, cannot be presented by its value on a free quark averaged over the proper probabilities of parton distributions. Therefore, the nonzero value of $g_2(x)$ can be obtained if we take into account parton interactions (or parton off-shellness). But the results of such calculations depend on the coordinate system (see below) and turn out to be physically unreasonable. So, in the conventional frame where the proton has pure z-component [2, 3]

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$$\int_0^1 g_2(x) dx = 0 \quad (2)$$

is not fulfilled.

It is convenient to analyze the parton model considering the old fashioned perturbation theory diagrams in the infinite momentum frame (IMF). In general, when we want to take into account parton interactions, in addition with naive parton model diagram of Fig. 3, the diagrams of fig.2 which contain $q\bar{q}$ -pairs creation or annihilation must be also taken into account.

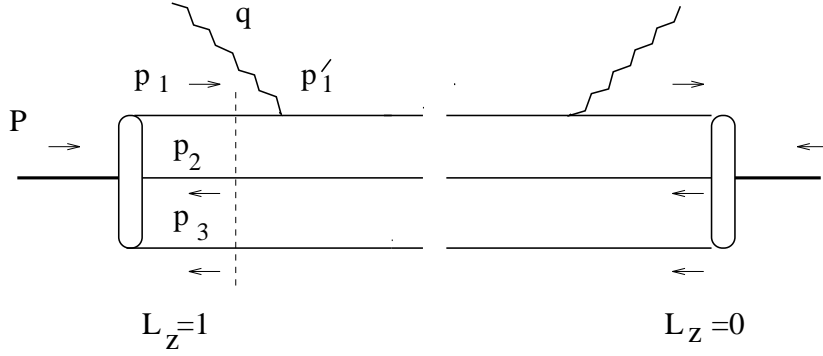


Fig. 1: The naive parton model diagram of deep inelastic scattering. The arrows denote nucleon and quarks helicities for helicity-flip amplitude which determine structure function $g_2(x)$

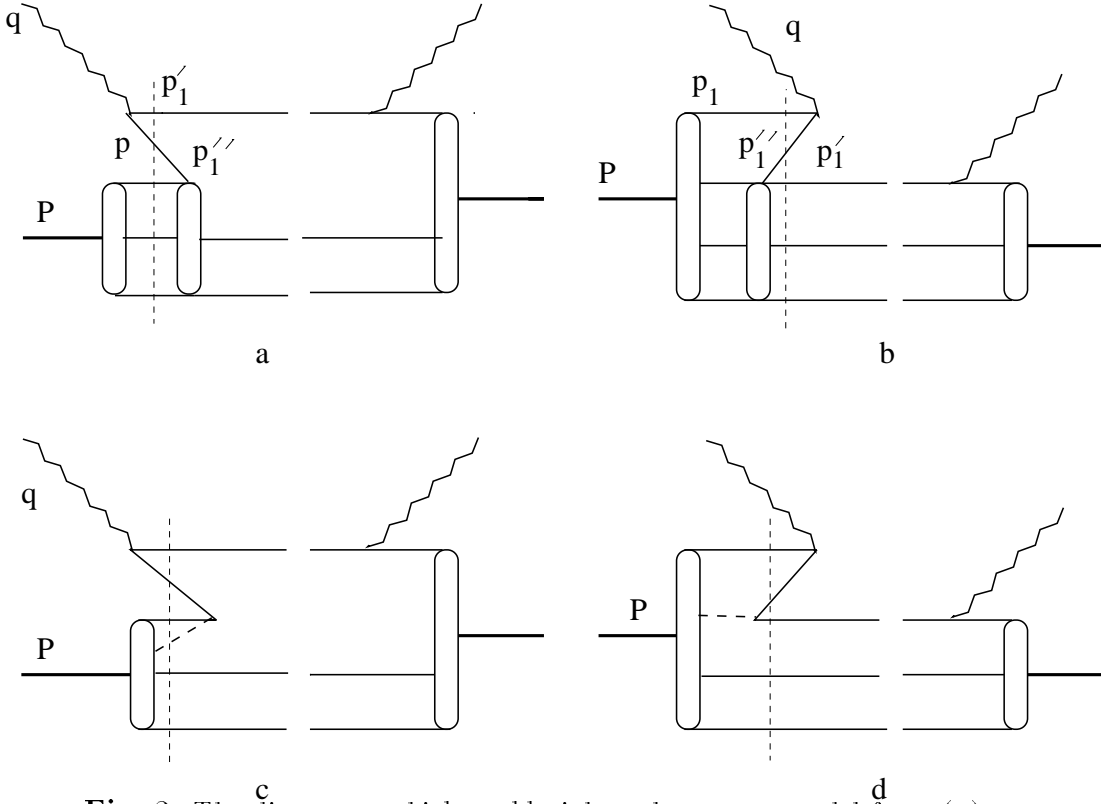


Fig. 2: The diagrams, which could violate the parton model for $g_2(x)$.

It is shown in this paper that such diagrams may contribute to $g_2(x)$ and violate the validity of parton model. We show that only in special IMF it is possible to get

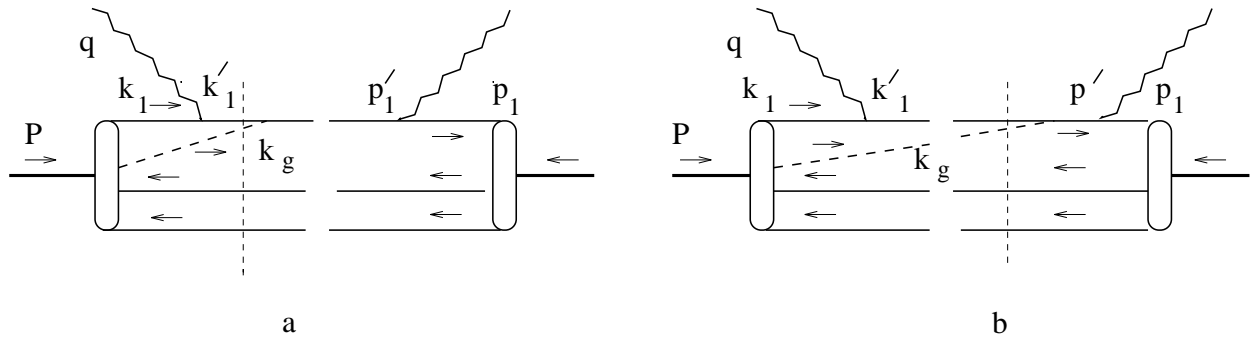


Fig. 3: The diagrams, which determine the contribution to $g_2(x)$ of Compton amplitude with different number of partons in the initial and final nucleon wave functions.

reed of such diagrams and to represent $g_2(x)$ in terms of quark-parton infinite momentum wave functions. I find that there are two phenomenologically independent contributions to $g_2(x)$. The first contribution is determined by the diagrams with the different values of quark orbital angular momentum projections in the initial and final nucleon wave functions (fig.1)¹. The second contribution is determined by Compton amplitudes with different number of gluons in the initial and final nucleon wave functions (fig.3). In the field theory these two contributions may be apparently connected.

The similar interpretation of $g_2(x)$ were discussed in Ref.5 in the framework of operator product expansion on "light cone".

In the second part of this paper I calculate the structure functions $g_1(x)$ and $g_2(x)$ in the relativistic quark model of ref.6. The results agree with the bag model calculations [7] and with QCD sum rule result [8, 9] and indicate a significant twist-3 contribution to $g_2(x)$ in the range of $x \leq 0.5$.

2 Quark-parton interpretation of $g_2(x)$

We start from the standart definition of the hadronic tensor:

$$\frac{M}{4\pi} W_{\mu\nu} = \sum_X (2\pi)^4 \langle p, s | J_\mu | X \rangle \langle X | J_\nu | p, s \rangle \delta(P + q - P_X) =$$

$$P_\mu P_\nu W_2 - M^2 W_1 g_{\mu\nu} + i M \epsilon_{\mu\nu\lambda\sigma} q^\lambda [M^2 s^\sigma G_1 + (P q s^\sigma - s q P^\sigma) G_2] \quad (3)$$

(disregarding terms proportional to q_μ or q_ν), where W_1, W_2, G_1 and G_2 are functions of q^2 and $\nu = Pq/M$. The nucleons are supposed in the same spin state described by s^μ . In the coordinate system (1) the spin average structure functions $F_1(x) = MW_1(x)$ and $F_2(x) = \nu W_2(x) = 2xF_2(x)$ are determined by symmetric part of the hadronic tensor W_{ij}^s ($i, j = 1, 2$). The spin dependent structure functions may be expressed through antisymmetric parts of W_{ij} and W_{i0} ($i = 1, 2$) as follows:

¹For simplicity, all diagrams are given for three quark state

$$\frac{1}{2\pi}W_{ij}^a = 2i\epsilon_{ij}g_1(x)\frac{s_0}{2P}$$

$$\frac{1}{2\pi}W_{i0}^a = i\epsilon_{ij}s_j(g_1(x) + g_2(x))\frac{2M}{P} \quad (4)$$

where the functions $g_1 = M^2\nu G_1$ and $g_2 = M\nu^2 G_2$ scales in the Bjorken limit. For quark momenta (which are defined on diagrams) we introduce the standart parameterizations:

$$\vec{p}_1 = x_1\vec{P} + \vec{p}_{1\perp}, \quad \vec{p}'_1 = -x_1\vec{P} + \vec{p}_{1\perp}, \quad \vec{p}''_1 = -x_1\vec{P} - \vec{p}_{1\perp},$$

$$\vec{p}_{1\perp}\vec{P} = 0 \quad (5)$$

In the coordinate system (1) the vertices of photon interactions with quarks (on fig.1) and with $q\bar{q}$ -pairs (on fig.2) at $P \rightarrow \infty$ behave as follows (i,j=1,2):

$$\bar{u}(p'_1)\gamma_i u(p_1) = 2Px_1\sigma_i\sigma_3, \quad \bar{u}(p'_1)\gamma_0 u(p_1) = 2(m + i\epsilon_{ik}\sigma_i p_{1k}),$$

$$\bar{u}(p'_1)\gamma_i v(p''_1) = 2(m\sigma_i + i\epsilon_{ij}p_{1j})\sigma_2, \quad \bar{u}(p'_1)\gamma_0 v(p''_1) = 2Px_1\sigma_3\sigma_2. \quad (6)$$

The amplitude of antiquark interaction with nucleon also may behave as P. Hence, the large energy denominators corresponding to dashed lines on diagrams of fig.2 may be compensated for W_{i0} (but not for W_{ij}) and these diagrams may contribute to $(g_1(x)+g_2(x))$. Thus, the naive parton model fails for $g_2(x)$ in the coordinate system (1). Let us consider now the special IMF where photon have pure transverse component at $P \rightarrow \infty$:

$$P_\mu = (E, P, 0, 0), \quad q_\mu = (\frac{q_\perp^2}{4Px}, \frac{-q_\perp^2}{4Px}, 0, 0). \quad (7)$$

In this frame the structure functions $g_1(x)$ and $g_2(x)$ are expressed through antisymmetric component of the hadronic tensor W_{i0}^a in the following form:

$$\frac{1}{4\pi}W_{i0}^a \frac{q_\perp^2}{x} = 2i\epsilon_{ij}q_j s_0 g_1(x)$$

$$\frac{1}{4\pi}W_{i0}^a \frac{q_\perp^4}{x^2} = 2i\epsilon_{ij}q_j \vec{s}\vec{g}_\perp M g_2(x) \quad (8)$$

We shall take longitudinally polarized nucleon to extract $g_1(x)$ and transversely polarized nucleon in \vec{q}_\perp direction to extract $g_2(x)$. But it is more transparent physically to represent $g_2(x)$ in terms of helicity amplitudes. In that language $g_2(x)$ corresponds to nucleon helicity-flip Compton amplitude. In the IMF (7) the vertices of photon interactions with quarks (on fig.1) and with $q\bar{q}$ -pairs (on fig.2) at $P \rightarrow \infty$ behave as follows (i=1,2):

$$\bar{u}(p'_1)\gamma_0 u(p_1) = 2Px_1, \quad \bar{u}(p'_1)\gamma_i u(p_1) = 2(p_{1i} + q_i + i\epsilon_{ik}q_k\sigma_3),$$

$$\bar{u}(p'_1)\gamma_0 v(p''_1) = \vec{\sigma}\vec{q}\sigma_3\sigma_2, \quad \bar{u}(p'_1)\gamma_i v(p''_1) = -2Px_1\sigma_i\sigma_2. \quad (9)$$

In the scaling limit the energy δ -function gives:

$$\delta(E + q_0 - E_X) = \frac{2Px_1}{q_\perp^2} \delta[x_1 - x(1 + \frac{2\vec{q}\vec{p}_{1\perp}}{q_\perp^2})]. \quad (10)$$

The contributions of diagrams of fig.2 to W_{i0} do not vanish at $P \rightarrow \infty$, this amplitude behaves as $W_{i0} \sim P/q_\perp^2$ in the scaling limit and these diagrams could contribute, in principle, to $g_2(x)$. But it is easy to see that W_{i0} do not depend on \vec{q}_\perp direction and hence cannot contain necessary structure: $\epsilon_{ij} q_j \vec{s} \vec{q}$. That means that such diagrams do not actually contribute to $g_2(x)$ and its correct value can be derived taking into account only diagram of fig.1, if we compare, for instance, terms proportional to $q_i q_j$ at both sides of (8).

At first sight the diagram of fig.1 also do not contribute to $g_2(x)$ because the corresponding value of W_{i0} has a wrong \vec{q}_\perp dependence and do not contain spin-flip terms. To obtain the nonzero value of $g_2(x)$ we have to take into account the second term of the expansion of δ - functions argument at $q_\perp^2 \rightarrow \infty$ in (10). Finally, the contribution of the diagram of Fig.1 may be presented in the form:

$$g_2^{(1)}(x) = \frac{d}{dx} \bar{g}(x),$$

$$2M\bar{g}(x) = \sum_r \int d\Gamma^n \delta(x - x_r) \sum_{s_1, \dots, s_n} \Psi_{s_i}^{*\dagger}(x_i, \vec{p}_{i\perp}) 2s_r Q_r^2 \Psi_{s_i}^\dagger(x_i, \vec{p}_{i\perp}) (p_{r\perp}^x + ip_{r\perp}^y), \quad (11)$$

where $d\Gamma^n$ is n-particle phase space:

$$d\Gamma^n = \frac{1}{x_n} \sum_{i=1}^{n-1} \frac{dx_i d\vec{p}_{i\perp}}{2x_i (2\pi)^3}, \quad (12)$$

Q_r and s_r denote charge and spin projection of active quark along z direction. The energy denominators are included into nucleon wave function:

$$\Psi_{s_i}(x_i, \vec{p}_{i\perp}) = \frac{\Gamma_{s_1 \dots s_n}(x_1, \vec{p}_{1\perp} \dots x_n, \vec{p}_{n\perp})}{2P(E - \sum_i E_i)} \quad (13)$$

where $\Gamma_{s_1 \dots s_n}(x_1, \vec{p}_{1\perp} \dots x_n, \vec{p}_{n\perp})$ is the nucleon-partons vertex.

The normalization condition of nucleon wave function (13) can be fixed, for instance, from normalization of nucleon electric formfactor $F_1(Q^2)$ at $Q^2 = 0$ in the quark parton model and has a following form:

$$\int d\Gamma^n \sum_{s_1, \dots, s_n} \Psi_{s_i}^*(x_i, \vec{p}_{i\perp}) \Psi_{s_i}(x_i, \vec{p}_{i\perp}) = 1 \quad (14)$$

(For more details of deriving Eqs.12-14 for three quark states see ref.5). In the integral over transverse momentum in (11) only terms which contains linear powers of transverse momenta in the final or initial wave functions contribute. Such terms can arise when the difference of angular orbital momentum projections of initial and final states is equal to unity, $\Delta\langle L_z \rangle = 1$; for such states the sum of parton helicities is not equal to nucleon helicity and hence nucleon spin-flip could take place as it is shown on the diagram (fig.1). The sum rule (2) will be fulfilled for (11)

if $\bar{g}(1) = \bar{g}(0) = 0$. The bound state wave functions vanish at $x = 0$ and $x = 1$ and the QCD-evolution should not violate the sum rule (2). So, if we suppose that quark-antiquark sea in the nucleon arise due to QCD-evolution and at low resolution scale nucleon can be considered as a bound state of finite number of constituents, the sum rule (2) will be fulfilled for (11).

The structure function $g_1(x)$ in the same notations has a following form:

$$2g_1(x) = \sum_r \int d\Gamma^n \delta(x - x_r) \sum_{s_1, \dots, s_n} \Psi_{s_i}^{*\dagger}(x_i, \vec{p}_{i\perp}) 2s_r Q_r^2 \Psi_{s_i}^\dagger(x_i, \vec{p}_{i\perp}). \quad (15)$$

Now consider the diagrams of fig.3. The momenta are defined on the diagrams:

$$\vec{p}_1 = x_1 \vec{P} + \vec{p}_{1\perp}, \quad \vec{k}_1 = y_1 \vec{P} + \vec{k}_{1\perp}, \quad \vec{k}_g = y_g \vec{P} + \vec{k}_{g\perp}, \quad (16)$$

The energy denominators corresponding to dashed lines on diagrams behave at $P \rightarrow \infty$ and $q_\perp \rightarrow \infty$ as

$$\frac{1}{2E_1(E + q_0 - \sum_i E_i)} = \mp \frac{x}{q_\perp^2(x_1 - y_1)}. \quad (17)$$

The upper sign in (16) corresponds to fig.3a, and bottom sign to fig.3b diagrams, respectively. The quark-gluon vertices on these diagrams behave as

$$\bar{u}(p'_1) \gamma_i u(k'_1) = q_i(x_1 + y_1) + i\sigma_3 \epsilon_{ik} q_k(x_1 - y_1) \quad (18)$$

for transverse gluons and these diagrams do contribute to $g_2(x)$ (but do not contribute to other structure functions). The diagrams with $q\bar{q}$ - pairs creations or annihilations (figs. 2c, 2d) do not contain necessary structure $\epsilon_{ij} q_j \vec{s} \vec{q}$ and do not contribute to $g_2(x)$. It is easy to understand that nucleon and gluon spins must be aligned in the same direction and the nucleon spin-flip takes place as it is demonstrated on the diagrams. We find the following result for these diagrams:

$$2Mg_2^{(2)}(x) = \sqrt{2x_r y_r} \sqrt{4\pi\alpha_s} \sum_r \int d\Gamma^n [\delta(x - x_r) - \delta(x - y_r)] \frac{dy_g d\vec{k}_{g\perp}}{(2\pi)^3 2y_r y_g^2} \times \sum_{s_1, \dots, s_n} (\Psi_{s_i}^{*\dagger}(x_i, \vec{p}_{i\perp}) \Psi_{s_i\uparrow}^\dagger(x_i, \vec{p}_{i\perp}) + \Psi_{s_i\downarrow}^{*\dagger}(x_i, \vec{p}_{i\perp}) \Psi_{s_i}^\dagger(x_i, \vec{p}_{i\perp})) 2s_r Q_r^2, \quad (19)$$

Here $\Psi_{s_i\lambda} = 1/2\lambda_a \psi_{s_i\lambda}^a$ (λ_a denotes Gell-Mann matrices) is the wave function of nucleon consisting of n-partons and one transverse gluon with helicity λ . The Burkhardt-Cottingham sum rule (2) is fulfilled for $g_2(x)$ due to cancelation of the contributions of fig.3a and fig.3b diagrams according to (17).

Thus, the structure function $g_2(x)$ in the parton model is determined by two physically independent mechanisms which do not contribute to other structure functions, so measuring $g_2(x)$ we can get essentially new information about quark-gluon structure of the nucleon, namely information about angular orbital momentum distribution and information about gluon distribution in the nucleon.

The direct connection of our results with the results of operator product expansion [5, 7] (see, also, Ref.10 and references therein) is not obvious and needs further

considerations. Note only, that as it easy to understand from Egs.11,15 $g_2^{(1)}(x)$ corresponds to twist-2 and twist-3 contributions, whereas $g_2^{(2)}(x)$ corresponds only to twist-3 contribution. The QCD evolution of $g_2^{(2)}(x)$ is, apparently, more complicated and differs from evolution of $g_2^{(1)}(x)$, so the structure function $g_2(x)$ cannot be evolved from its value at some low Q^2 , in accordance with results of operators product expansion [6, 7].

3 Numerical calculations of $g_1(x)$ and $g_2(x)$ in relativistic quark model

In order to estimate the role of twist-3 contribution and to understand in details the physical interpretation of $g_2(x)$ it is worth to calculate this function in a specific model. In this section the results of such calculation in the relativistic quark model (RQM) of Ref.6 are presented. We actually assume that at low virtualities nucleon consists of three valence constituent quarks only and do not discuss the contributions of diagrams of fig.3. In such a simple model it is impossible to reproduce the the correct Regge behavior at $x \rightarrow 1$. So, our results are expected to be reasonable in the range of not small values of x ($x \leq 0.2 - 0.3$), where the contributions of nonperturbative quark-antiquark pairs are expected to be small. The results obtained must be evaluated into the range of experimental values of Q^2 . In general, it is impossible to write down the simple evolution equation for twist-3 contribution to $g_2(x)$ which would allow to connect this function at different values of Q^2 [10, 11]. Nevertheless in Ref.12 it was found that such approximate equation (which becomes exact in $N \rightarrow \infty$ limit) exist and utilizing this result it is possible to evaluate our results into the range of experimental momentum transfer.

In RQM the spin dependent structure functions $g_1(x)$ and $g_2(x)$ are determined by Egs.11,15 with $n=3$. The nucleon wave functions (13) in the IMF have following form [6]²:

$$\Psi_{s_1 s_2 s_3}^s(x_i, p_{i\perp}) = \Phi(M_0^2) U_{s_1 s'_1}(x_1, p_{1\perp}) U_{s_2 s'_2}(x_2, p_{2\perp}) U_{s_3 s'_3}(x_3, p_{3\perp}) \chi_{s'_1 s'_2 s'_3}^s \quad (20)$$

where $\Phi(M_0^2)$ is a radial part which supposed to depend only of one argument - invariant mass of the system of quarks, composing nucleon [6],

$$M_0^2 = \sum_{i=1}^3 \frac{p_{i\perp}^2 + m^2}{x_i}, \quad (21)$$

where m stands for constituent quark mass; $\chi_{s'_1 s'_2 s'_3}^s$ is the spin orbital part of wave function which is supposed to coincide with nonrelativistic wave functions of naive quark model. The Melosh matrices,

²Different variants of relativization of quark model differ, in general, by explicite form of Melosh matrices, see e.g.,[13] and references therein.

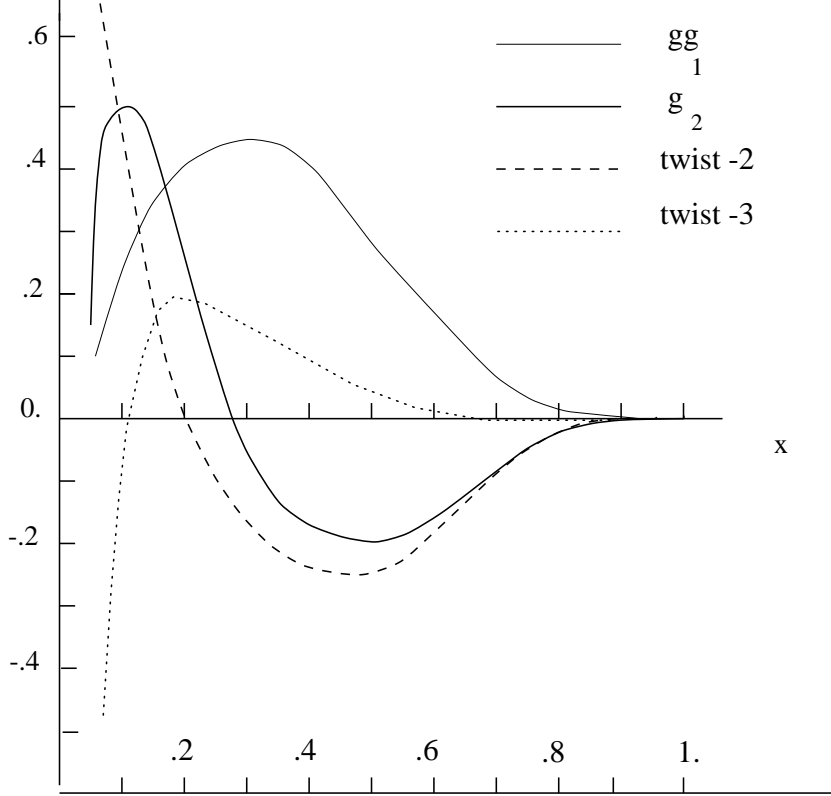


Fig. 4: The results of numerical calculations of $g_2(x)$ and $g_2(x)$ in relativistic quark model.

$$U(x_i, p_{i\perp}) = \frac{m + M_0 x_i + i \epsilon_{mn} \sigma_m (p_{i\perp})_n}{(m + M_0 x_i)^2 + p_{i\perp}^2} \quad (22)$$

determine the transformation of $\chi_{s_1 s_2 s_3}^s$ functions from rest frame into IMF. In Ref.6 a good description of nucleon static parameters were obtained under assumption that nucleon wave function is pure $[56.0^+]$ representation of SU(6) group. For radial wave function it was supposed the following form:

$$\Phi(M_0^2) = N \exp(-M_0^2/6\alpha^2) \quad (23)$$

The fitting parameters, α and m turned out to be [6]:

$$\alpha = 380 \pm 60 \text{ Mev}, m = 270 \pm 30 \text{ Mev} \quad (24)$$

Note, that parameter α characterizes the mean square momentum of quark in nucleon: $p_{\perp}^2 \simeq 2/3\alpha^2$. It is very important to note that due to Melosh transformation in 20 the nucleon IMF wave function depends on quark transvers momentum even for pure $[56.0^-]$ representation. This actually means that that due to Melosh transformation in pure SU(6) wave function in IMF arises the admixture of states with nonzero value of angular orbital momentum. It makes it possible to derive the nonzero value of $g_2(x)$. We find the following results for spin dependent structure functions:

$$2g_1^p(x) = \frac{4Q_u^2 - Q_d^2}{3} \int d\Gamma^3 \delta(x - x_1) \left[1 - \frac{2p_{1\perp}^2}{(m + M_0 x_1)^2 + p_{1\perp}^2} \right] \quad (25)$$

$$2Mg_2^p(x) = \frac{4Q_u^2 - Q_d^2}{3} \frac{d}{dx} \int d\Gamma^3 \delta(x - x_1) \frac{p_{1\perp}^2 (m + M_0 x_1)}{(m + M_0 x_1)^2 + p_{1\perp}^2} \quad (26)$$

For the neutron structure functions the replacement $Q_u \leftrightarrow Q_d$ must be inserted in (25,26) and we find:

$$g_1^n(x) = g_2^n(x) = 0 \quad (27)$$

This result qualitatively agrees with low experimental value of $g_1(x)$. The nonzero values of $g_1(x)$ and $g_2(x)$ can be obtained if we take into account the admixture of high multiplets in the nucleon wave function.

The nucleon β -decay coupling

$$g_A = 6 \int (g_1^p(x) - g_1^n(x)) dx \simeq 1.22 \quad (28)$$

is well described in the model.

The part of nucleon spin carried by quarks in IMF is given by

$$2\langle S_z \rangle = \int d\Gamma^3 \left[1 - \frac{2p_{1\perp}^2}{(m + M_0 x_1)^2 + p_{1\perp}^2} \right] \simeq 0.73 \quad (29)$$

and turned out to be less than unity even for pure SU(6) nucleon wave function (see also Ref.14). The missing part of nucleon spin is carried by angular orbital momentum L_z which arise due to Melosh transformations. It is easy to check that $\langle L_z \rangle$ exactly correspond to the second term in square brackets in (30). But this interesting relativistic effect cannot account for the spin deficit which follows from experimental data. Keeping in mind these results (29-30) it is natural to assume that the considered model more successfully can be applied in flavour non-singlet channel. This is, apparently, connected with possible cancelation of unknown contributions which could be essential in singlet channel. So, for comparison with experiment or with predictions of other models it is more reasonable to consider the differences of proton and neutron structure functions.

The results of numerical calculations are presented on Fig.4. The twist-3 contribution to $g_2(x)$ was obtained by subtracting from $g_2(x)$ (27) the twist-2 contribution which is determined by $g_1(x)$ [15]:

$$g_2^{tw.2}(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy \quad (30)$$

Our results are very close to the bag model calculations of Ref.7 and indicate that twist-3 contribution is not small. Remind that these results correspond to low resolution scale, of order of typical constituent quark virtualities in the nucleon. The evolution to the experimental values of Q^2 will increase our predictions considerably (about 5-8 times) in the range of $x \geq 0.5$.

The second moments of twist-3 contribution to $g_2(x)$ were calculated in Refs.8,9 in framework of QCD sum rules:

$$M_{p-n}^{(2)} = \int (g_2^p - g_2^n) x^2 dx = 0.008 \pm 0.004 \quad (31)$$

$$M_{p+n}^{(2)} = \int (g_2^p + g_2^n) x^2 dx = -0.009 \pm 0.004 \quad (32)$$

Our result for difference $g_2^p(x) - g_2^n(x)$

$$M_{p-n}^{(2)} \simeq 0.002 \quad (33)$$

agrees with (31) by sign and by order of magnitude.

4 Conclusion

In the special infinite momentum frame the structure function $g_2(x)$ has a reasonable interpretation in terms of nucleon parton wave function (but not in terms of probabilities). We have two contribution which are phenomenologically independent. The first contribution corresponds to Compton diagrams with different values of angular orbital momenta projections in the initial and final nucleon wave functions. This contribution is proportional to quark transvers momentum and corresponds to twist-2 and twist-3 contributions in terms of OPE. The second contribution is determined by Compton diagrams with different number of gluons in initial and final nucleon wave functions and correspond only to twist-3.

According to independent estimates in different models the function $g_2(x)$ and the twist-3 contribution to $g_2(x)$, which really contains new information on quark-gluon interaction in nucleon are not small. So, the projected measurements of this function are reasonable and will provide new information on nucleon structure.

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